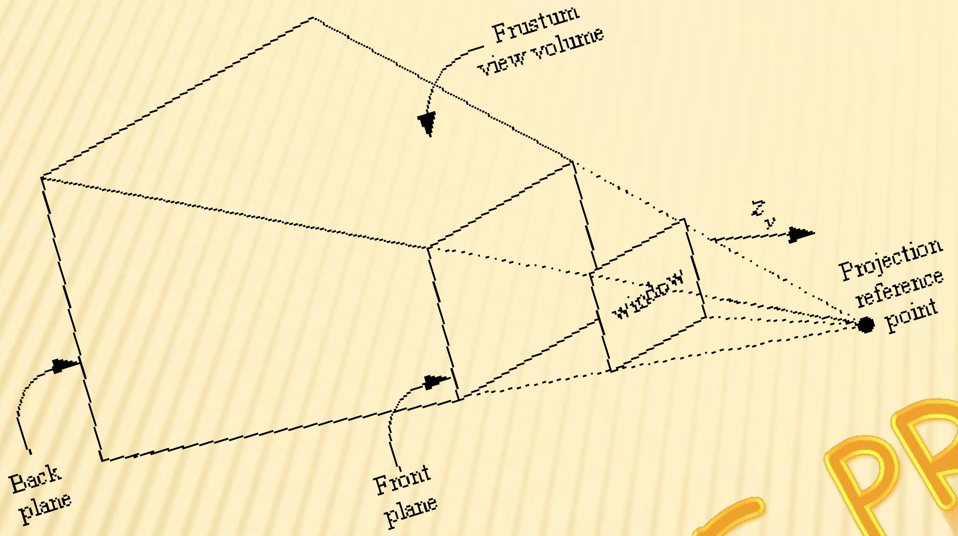
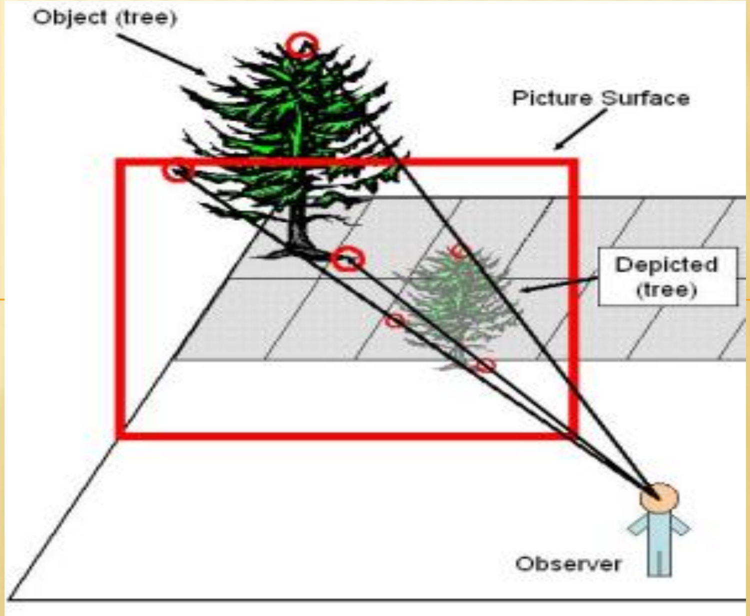
PERSPECTIVE PROJECTION



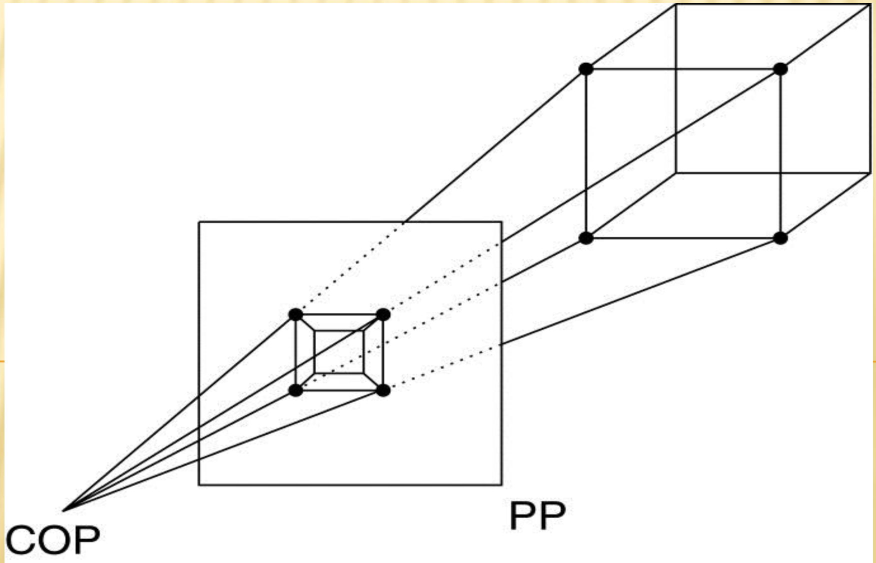


Projection

Projections transform points in *n*-space to *m*-space, where *m*<*n*.

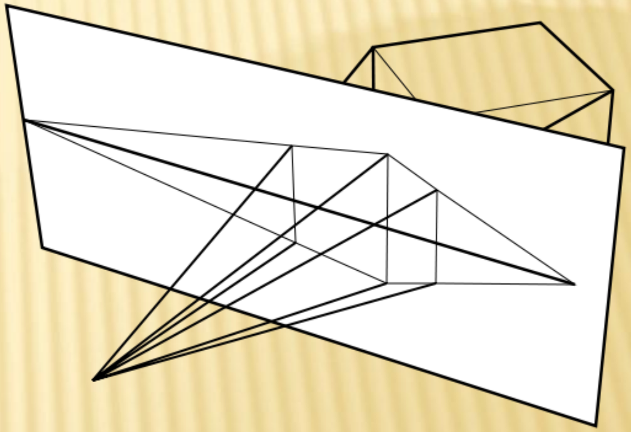
Projection is 'formed' on the **view plane** (planar geometric projection) rays (projectors) projected from the **center of projection** pass through each point of the models and intersect projection plane.

In 3-D, we map points from 3-space to the **projection plane** (PP) (image plane) along **projectors** (viewing rays) emanating from the center of projection (COP):

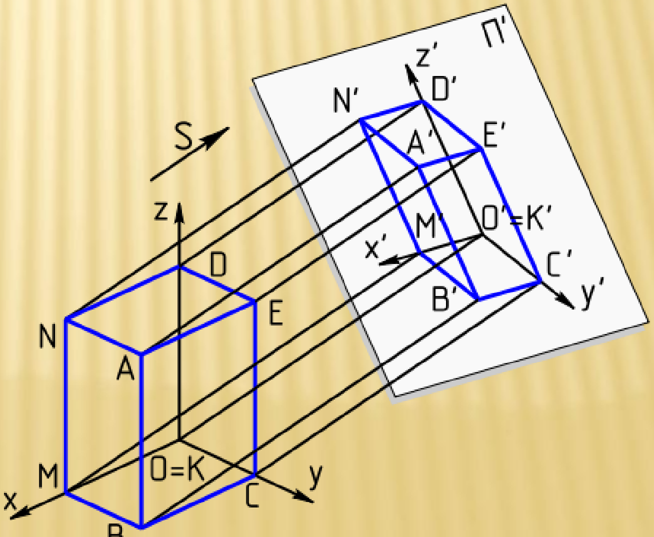


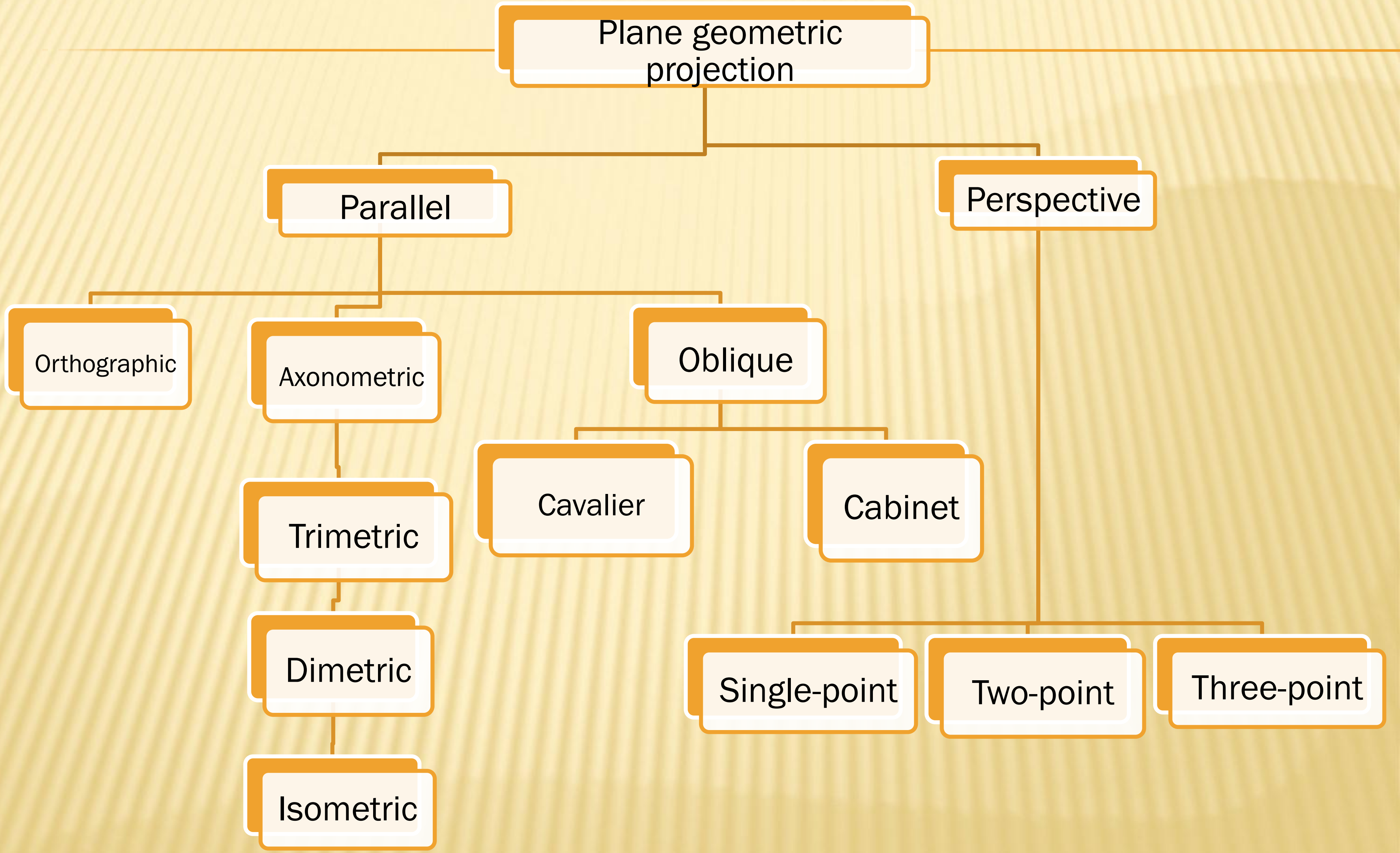
TYPES OF PROJECTION

- There are two basic types of projections:
  - Perspective – center of projection is located at a finite point in three space



  - Parallel – center of projection is located at infinity, all the projectors are parallel



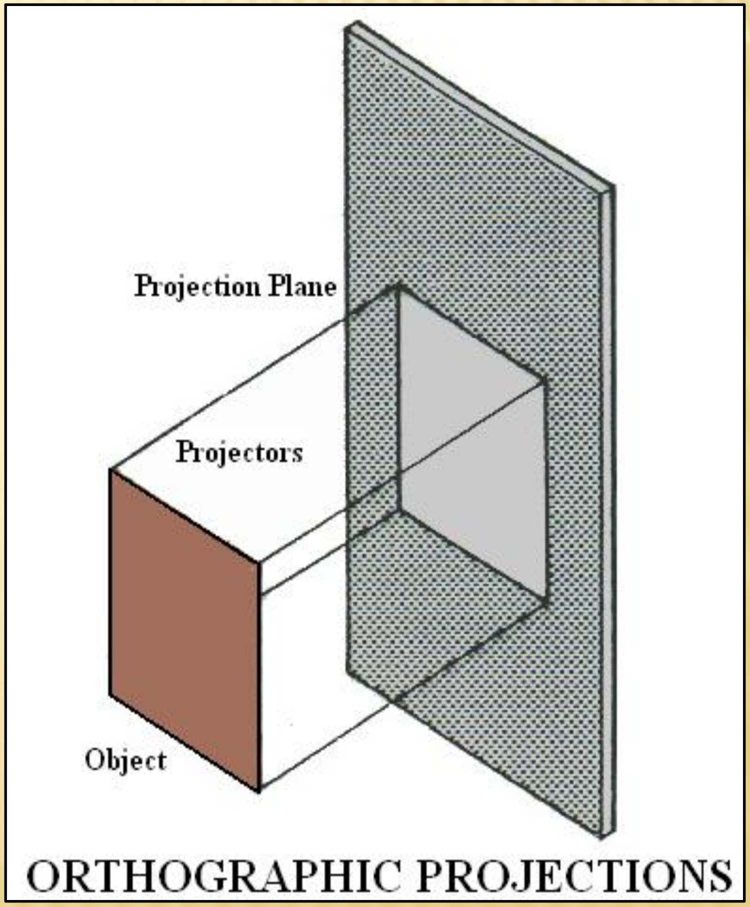

Plane geometric projection
Parallel
Perspective
Orthographic
Axonometric
Oblique
Cavalier
Cabinet
Trimetric
Dimetric
Isometric
Single-point
Two-point
Three-point

PARALLEL PROJECTION

- center of projection infinitely far from view plane
- projectors will be parallel to each other
- need to define the **direction of projection** (vector)
- 3 sub-types
  - Orthographic - direction of projection is normal to view plane
  - Axonometric – constructed by manipulating object using rotations and translations
  - Oblique - direction of projection not normal to view plane
- better for drafting / CAD applications

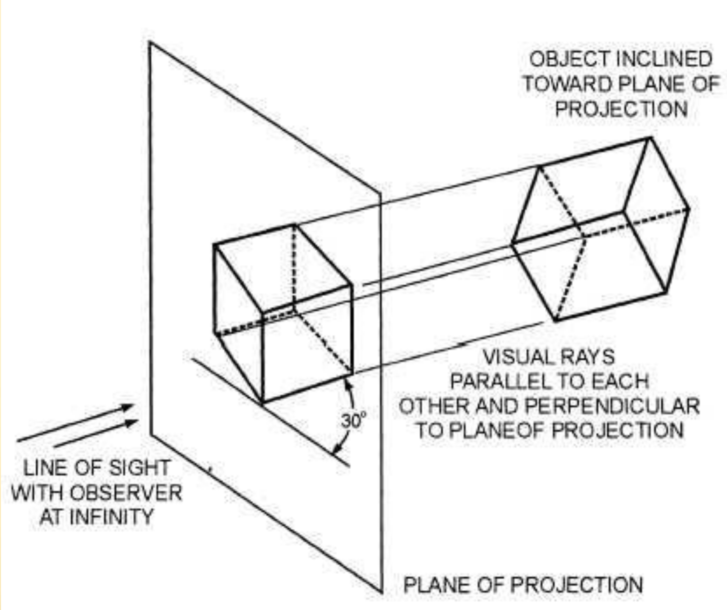
ORTHOGRAPHIC PROJECTIONS

- Orthographic projections are drawings where the projectors, the observer or station point remain parallel to each other and perpendicular to the plane of projection.
- Orthographic projections are further subdivided into axonom etric projections and multi-view projections.
- Effective in technical representation of objects



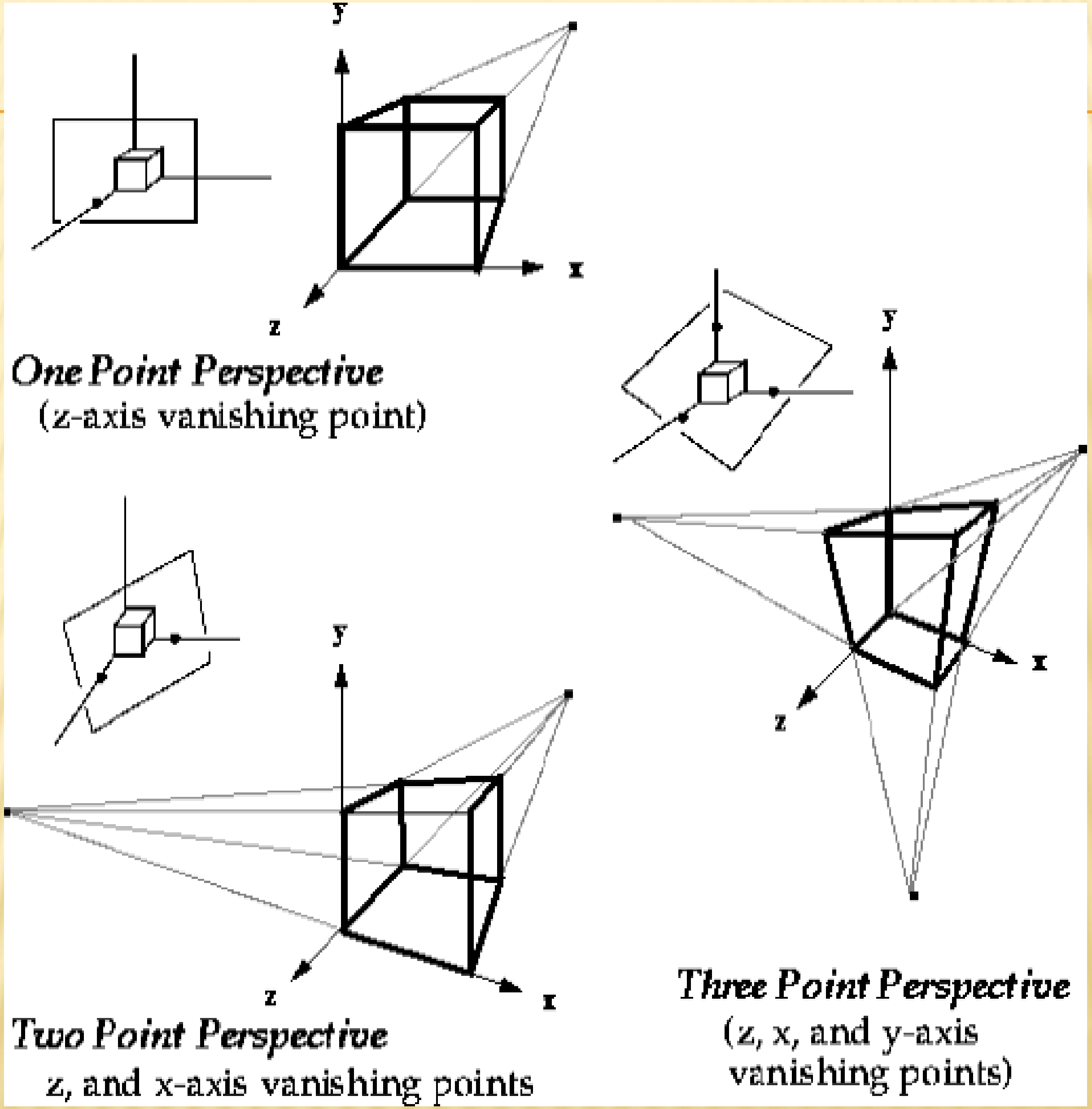
AXONOMETRIC

- The observer is at infinity & the projectors are parallel to each other and perpendicular to the plane of projection.
- A key feature of axonometric projections is that the object is inclined toward the plane of projection showing all three surfaces in one view.
- The length of the lines, sizes of the angles, and proportions of the object varies according to the amount of angle between the object and the plane of projection.



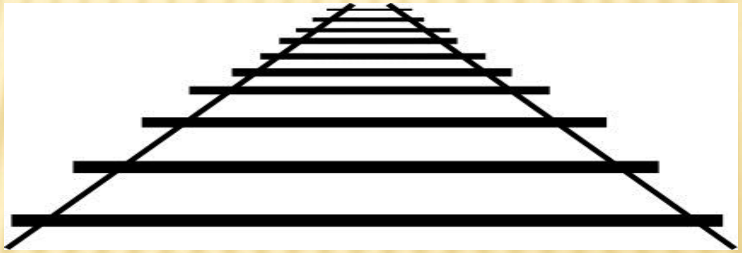
PERSPECTIVE PROJECTION

- center of projection finitely far from view plane
- projectors will not be parallel to each other
- need to define the location of the **center of projection** (point)
- classified into 1, 2, or 3-point perspective
- more visually realistic - has perspective foreshortening (objects further away appear smaller)

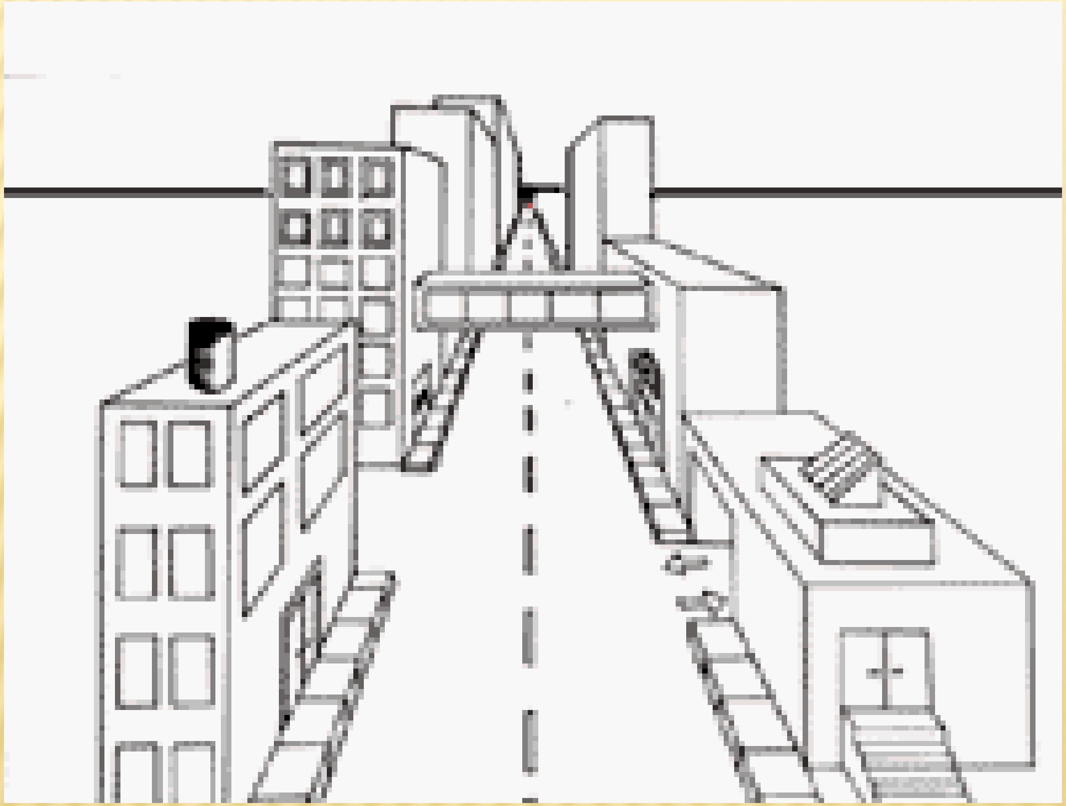

y
x
z
One Point Perspective
(z-axis vanishing point)
y
x
z
Three Point Perspective
(z, x, and y-axis
vanishing points)
y
x
z
Two Point Perspective
z, and x-axis vanishing points

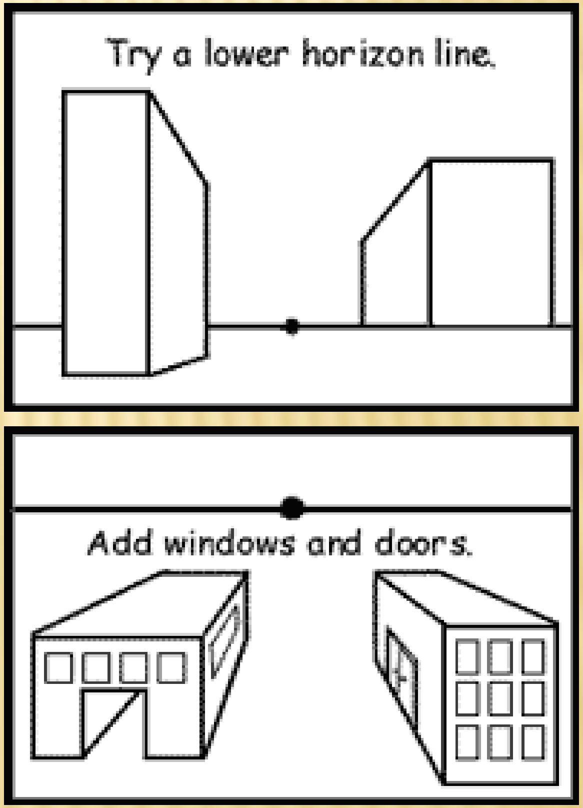
VANISHING POINTS

- A **vanishing point** is a point in the picture plane that is the intersection of the projections (or drawings) of a set of parallel lines in space on to the picture plane.



PERSPECTIVE TRANSFORMATIONS





SINGLE – POINT

- If any one of the first 3 values of the last column of a 4x4 transformation matrix is non-zero, then a perspective transformation occurs.
- After the transformation, a perspective projection is performed by concatenating the perspective transformation with an orthographic projection along a plane.

A SINGLE POINT PERSPECTIVE

A single point perspective transformation with respect to z axis

axis

$$[x \quad y \quad z \quad 1]\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & r\\0 & 0 & 0 & 1\end{bmatrix} = [x \quad y \quad z \quad (rz+1)]$$

$$[x^* \quad y^* \quad z^* \quad 1] = \left[\frac{x}{rz+1} \quad \frac{y}{rz+1} \quad \frac{z}{rz+1} \quad 1\right]$$

Now the perspective projection is obtained by concatenating the orthographic projection matrix

$$[T] = [P_t][P_z] = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & r\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & r\\0 & 0 & 0 & 1\end{bmatrix}$$

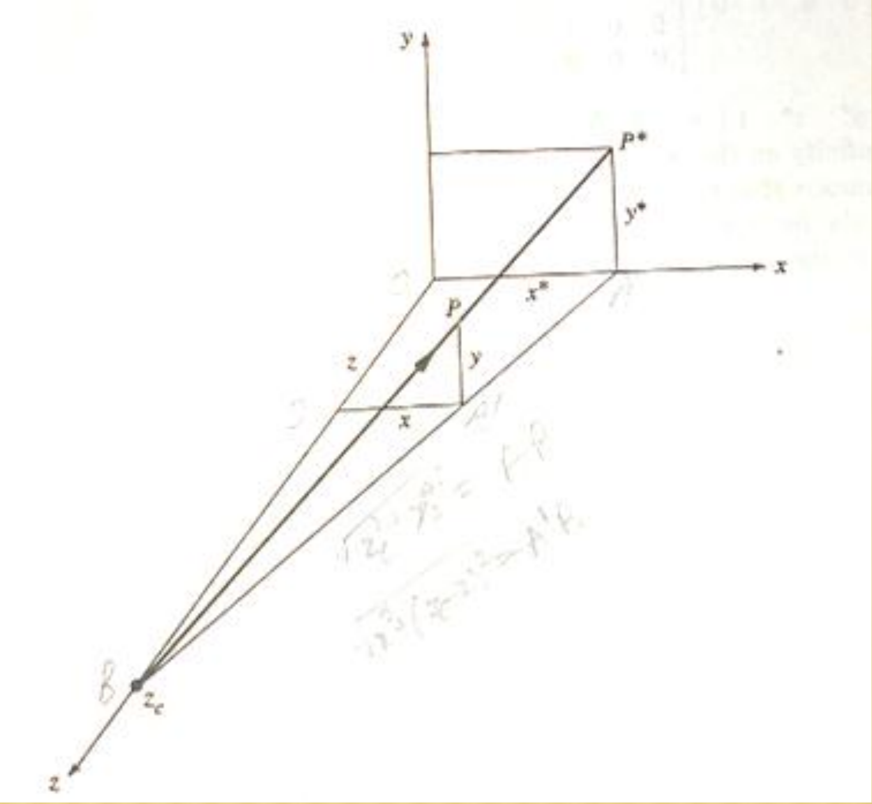
$$[x^* \quad y^* \quad z^* \quad 1] = \left[\frac{x}{rz+1} \quad \frac{y}{rz+1} \quad 0 \quad 1\right]$$

THE PERSPECTIVE PROJECTION OF A POINT

- Consider the following figure where using similar triangles we can write the transformed coordinates as:

$$\frac{x^*}{z_c} = \frac{x}{z_c - z} \qquad \Rightarrow x^* = \frac{x}{1 - \frac{z}{z_c}}$$

$$\frac{y^*}{\sqrt{x^{*2} + z_c^{*2}}} = \frac{y}{\sqrt{x^2 + (z_c - z)^2}} \qquad \Rightarrow y^* = \frac{y}{1 - \frac{z}{z_c}}$$



Let,

$$r = -\frac{1}{z_c} \Rightarrow \frac{x}{1+rz};\frac{y}{1+rz}$$

The origin is unaffected. If the plane of projection passes through the object, then that section of the object is shown at true size and true shape

THE PERSPECTIVE PROJECTION OF A POINT

A single point perspective transformation with respect to xaxis and y-axis are given below respectively

$$[T]=[P_t][P_z]=\begin{bmatrix}1&0&0&p\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0&p\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}x^* & y^* & z^* & 1\end{bmatrix}=\begin{bmatrix}\dfrac{x}{px+1} & \dfrac{y}{px+1} & 0 & 1\end{bmatrix}$$

$$[T]=[P_t][P_z]=\begin{bmatrix}1&0&0&0\\0&1&0&q\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&q\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

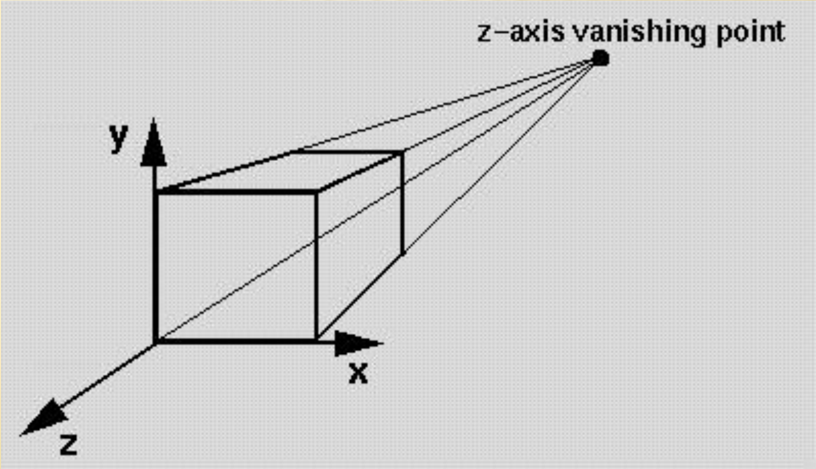
$$\begin{bmatrix}x^* & y^* & z^* & 1\end{bmatrix}=\begin{bmatrix}\dfrac{x}{qy+1} & \dfrac{y}{qy+1} & 0 & 1\end{bmatrix}$$

CONDITIONS FOR ONE POINT PERSPECTIVE

- For One Point perspective projection the object must have two of its primary axis' placed parallel to the picture plane. The following properties are achieved where one main set of parallel lines are receding away from the spectator thus giving us one vanishing point.

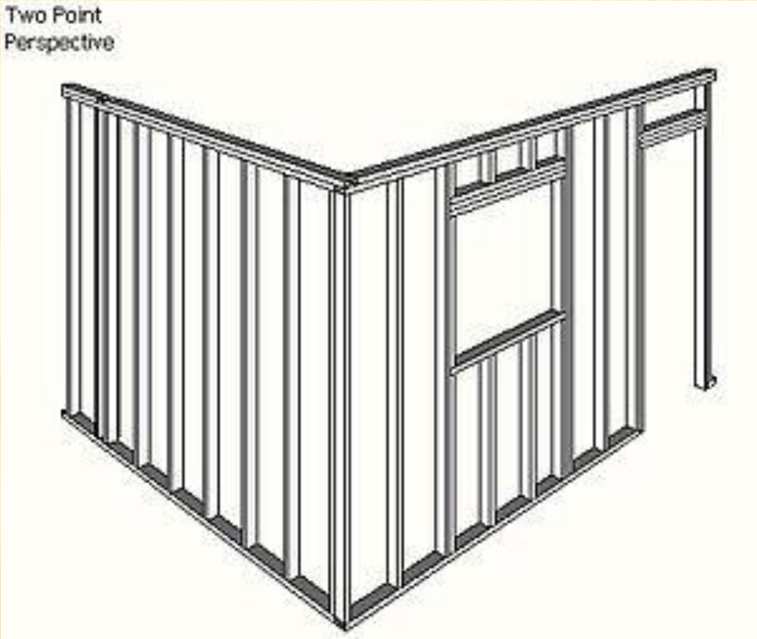
RULES OF ONE POINT PERSPECTIVE

- Horizontal lines parallel to the picture plane and remain horizontal.
- Vertical lines remain vertical.
- The orthogonal lines formed from the corners diverge to the vanishing point.
- The vanishing point is located in front of the spectator .



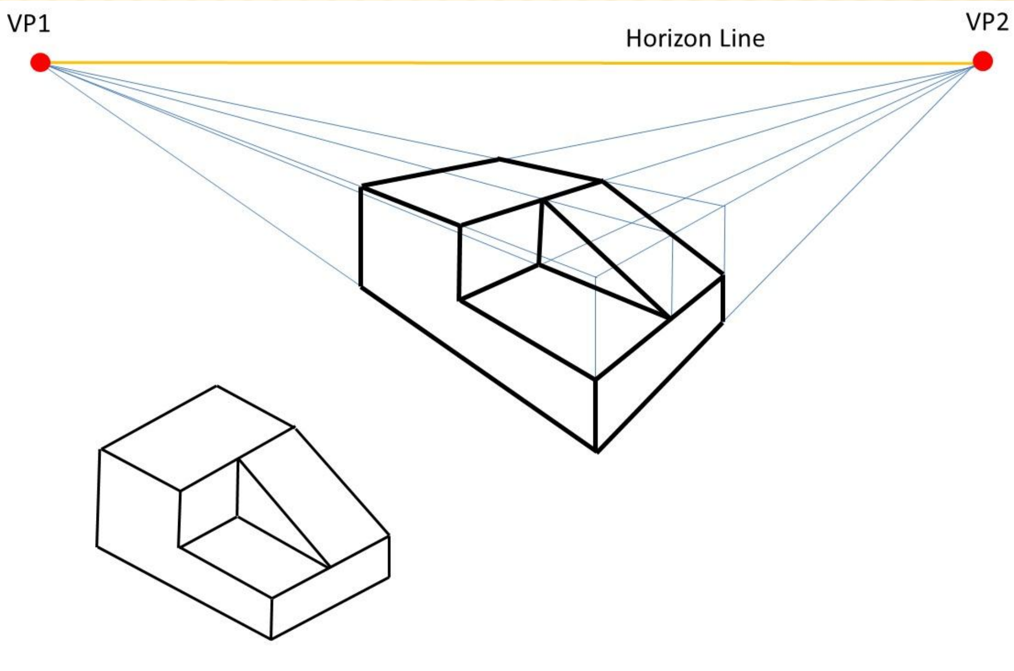
TWO - POINT

- It contains two vanishing points on the horizon line
- Object is at angle with picture plane, but vertical edges are parallel to picture plane



CONDITIONS FOR TWO POINT PERSPECTIVE

- For Two Point perspective the object is orientated so that the vertical edges are parallel to the picture plane and all other edges are inclined away from the spectator thus the receding lines converge on two separate vanishing points.

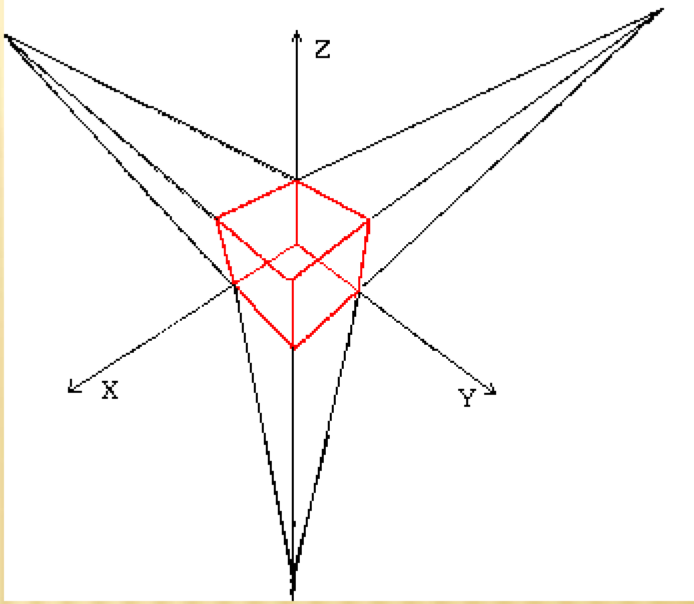


RULES OF TWO POINT PERSPECTIVE DRAWING.

- To find the vanishing points, lines of sight are drawn parallel to the surfaces of the object until they cross the picture plane.
- The object to be viewed is rotated so that all the sides are at an angle to the picture plane.
- All vertical lines retract back to their respective vanishing points
- Horizontal lines remain horizontal

THREE - POINT

- Three-point perspective is usually used for buildings seen from above (or below). In addition to the two vanishing points from before, one for each wall, there is now one for how those walls recede into the ground. This third vanishing point will be below the ground.



Example 3–26 Principal Vanishing Points by Transformation

Recalling Ex. 3–24 the concatenated complete transformation was

$$[T] = \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transforming the points at infinity on the x-, y- and z-axes yields

$$[VP][T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6.142 & 2.5 & 0 & 1 \\ 0 & -2.5 & 0 & 1 \\ 2.04 & 2.5 & 0 & 1 \end{bmatrix}$$

Example 3-27 Trace Points by Transformation

Consider the simple triangular prism shown in Fig. 3-38a. The position vectors for the prism are

$$[X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0.5 & 0.5 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0.5 & 0.5 & 0 & 1 \end{bmatrix}$$

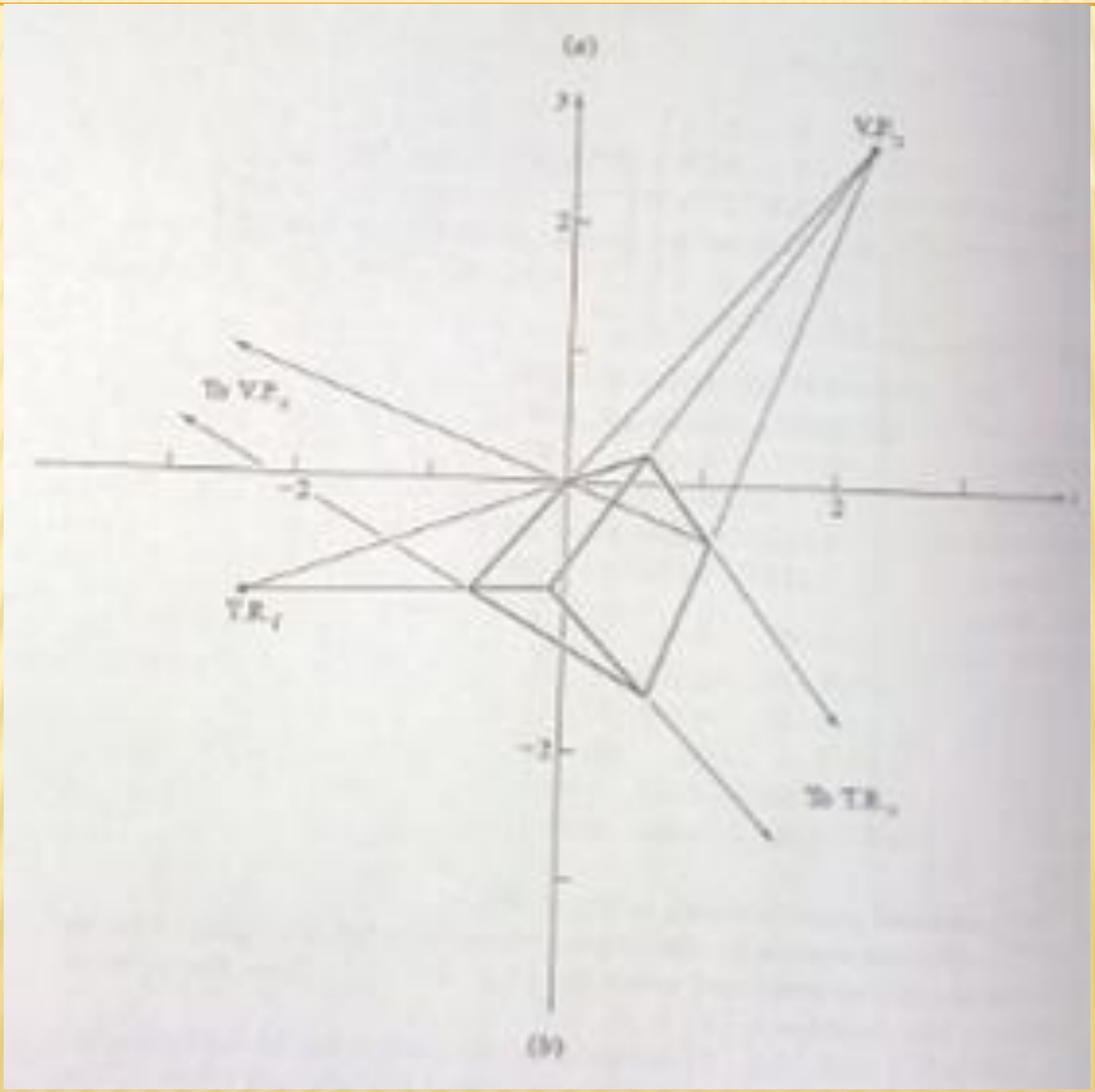
Applying the concatenated transformation of Ex. 3-24 yields transformed position vectors

$$\begin{aligned}
[X^*] &= [X][T] \\
&= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0.5 & 0.5 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0.5 & 0.5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -0.5 & -0.612 & 0 & 0.755 \\ 0.366 & -0.966 & 0 & 0.614 \\ -0.067 & -0.436 & 0 & 0.543 \\ 0 & 0 & 0 & 1 \\ 0.866 & -0.354 & 0 & 0.859 \\ 0.433 & 0.177 & 0 & 0.788 \end{bmatrix} \\
&= \begin{bmatrix} -0.662 & -0.811 & 0 & 1 \\ 0.596 & -1.574 & 0 & 1 \\ -0.123 & -0.802 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1.009 & -0.412 & 0 & 1 \\ 0.55 & 0.224 & 0 & 1 \end{bmatrix}
\end{aligned}$$

The transformed prism is shown in Fig. 3-38b.

The direction cosines for the inclined edges of the left-hand plane forming the top of the untransformed prism are $[0.5 \quad 0.5 \quad 0]$. Thus, the point at infinity in this direction is $[1 \quad 1 \quad 0 \quad 0]$.

Similarly, $[-0.5 \quad 0.5 \quad 0]$ are the direction cosines for the inclined edges of the right-hand plane forming the top of the untransformed prism. Thus, the point at infinity in this direction is $[-1 \quad 1 \quad 0 \quad 0]$.


(a)
V.P.
To V.P.
T.R.
To T.R.
(b)

Transforming these infinite points along with those for the principal axes yields

$$[VP][T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0.866 & 0.354 & 0 & -0.424 \\ -0.866 & 1.061 & 0 & -0.141 \end{bmatrix}$$

$$= \begin{array}{cl} \begin{bmatrix} -6.142 & 2.5 & 0 & 1 \\ 0 & -2.5 & 0 & 1 \\ 2.041 & 2.5 & 0 & 1 \\ -2.041 & -0.833 & 0 & 1 \\ 6.142 & -7.5 & 0 & 1 \end{bmatrix} & \begin{matrix} VP_x \\ VP_y \\ VP_z \\ TP_1 \\ TP_2 \end{matrix} \end{array}$$

The vanishing and trace points are also shown in Fig. 3-36b. Notice that as expected VP_x, VP_y and VP_z are the same as found in Ex. 3-26.